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Classical physics of magnetic moment

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Abstract Excitation of magneto-static field in matter is essentially due to orientation and distribution of magnetic moment of material charges, whatever be its apparent explanation classically and phenomenologically Classical physics of magnetic moment which is a primary source of magnetization in matter, has been developed systematically for various applications, including particularly those of determination of the DC moments in problems of wave plasma interactions as well as wave-wave interactions in plasma

Keywords Magnetic moment field, inverse Faraday effect, magnetization

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1 Introduction

Primary sources of excitation of electromagnetic field are charges and electric currents; and, secondary sources are polarizable and magnetizable matter. Matter is polarized by Coulomb field and magnetized by electric currents. Charges and electric currents in matter generate magnetic moment physically due to bending of their motion. The formula for magnetic moment is given by fundamental laws of electrodynamics of Biot-Savart and Ampere, which state that electric currents in loops generate axial magnetostatic field. Loop motion means a series of changes of direction of motion of particles of fluid elements. These loops are not necessarily circular in finite regions. Any infinitesimal change in the direction of motion, due to disturbances which are not symmetric in all directions, excites instantaneously the magnetic field. Asymmetry in motion means a gain or loss of angular moment, which is proportional to the magnetic moment of the charged particles.

Natural rotation of a body depends on the orientation and distribution of angular momentum of its smaller parts. Magnetization of a body is due to the spin of its charges, which generate ordered magnetic moments. So, this field depends on the orientation and distribution of the magnetic moment of its charge. Quantum mechanical treatment of physics of magnetization in matter depends on this physics [1]. But, classical theory for excitation of magnetization in matter, generally, is considered without the help of concepts of magnetic moment. Apparently, magnetization depends on the nature of the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ (which implicitly depend on the nature of electric current including chagre motion) and other conditions specifying problems. Determination of these currents is not necessary for practical purposes. For excitation of magnetic field in the important cases (cases of dynamo field of planets and stars, Hall current field in MHD power generation, thermal gradient field in inertial confinement for fusion reaction for generation of power), the classical investigations do not depend on the physics of magnetic moment. Classical dynamo theories for different models of equilibrium configuration, matter, motion and fields depends on the *a priori* existence of seed magnetic field.

Sources for magnetic field are primary if *a priori* existence of a seed field is not necessary. *A priori* existence of a seed field is necessary for secondary sources. The

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magnetic moment generation process, the thermal gradient sources, the field of ponderomotive origin is examples of primary sources. The dynamo field and the field from Hall current are examples of secondary sources. An elliptically polarized wave in a fully ionized, unmagnetized and non-collissional plasma excites the DC magnetic moment density parallel to the direction of wave propagation. This findings ensure that magnetic moment is a primary source of generation of magnetic field. This elliptically polarized field in a plasma, can be a locally developed thermal field which is possible in space bodies radiating EM fields.

Magnetization in a body depends on orientation and distribution of magnetic moment of charges in it. So, there is nothing like secondary sources and seed magnetic field for excitation of magnetization in matter. Magnetic field, including DC fields, is exclusively generated by magnetic moment of charges in wave affected matter. This contradicts the astrophysical assumption of primeval magnetic fields by the dynamo theories which require the *a priori* existence of seed magnetic field [2,3].

The principal sources of magnetic moment of an atom are : (i) the electron spin, (ii) the orbital angular momentum of an electron about the nucleus, and (iii) the applied field induced change in the angular momentum. Classically, the angular momentum and magnetic moment of electrons are related even in the atomic scale, only for the orbital angular momentum. The spin of magnetic materials may be treated as the classical angular momentum vector [4]. The intrinsic (spin) magnetic moment of atomic electrons, generates dipole fields which vary rapidly on the scale of atomic dimensions. Magnetic substances have magnetic properties of spin systems, and also have electrical conductivity and dielectric polarization. The collective behavior of spin systems excites electro-kinetic spin waves in magnetic substances. Spin waves are quasi particles called magnons. Like plasmons and phonons, these are bosons.

Plasma, the fourth state of matter, does not have the thermal limit of Curie point of temperature T_c for its magnetization, because the long-range collective phenomena in a plasma are not destroyed with increase of temperature. In magnetic materials below T_c, orientation and distribution of spin angular momentum of electrons explain its magnetic momentum. Above T_c, thermal agitation destroys the line of magnetic moment parallel to each other. So, the field vanishes. The Curie temperature T_c separates the disordered paramagnetic phase at $T > T_c$ from the ordered ferromagnetic phase at $T < T_c$. Also the latent heat of transition from one state to another (say from liquid state to gaseous state) is not important in physics of plasmas. In spite of these distinction between plasma and magnetic materials, the magnetic like behaviour of plasma can be studied.

The magnetic moment of hydrogen nucleus is about 10^{-3} times smaller than the electron magnetic moment. So, interaction of magnetic moment of ionic cores with the magnetic moment of the surrounding electrons, though weak, is studied, assuming that ion dynamics is not negligible. The intrinsic (spin) magnetic moment of atomic electrons, generates fields which vary rapidly in the scale of atomic dimensions. So, the spin angular momentum and some other effects are studied quantum mechanically in the atomic scale.

Originally, this magnetization was regarded as the inverse of Faraday rotation from birefringence of waves of right and left circular polarization propagating parallel to the direction of an ambient magnetostatic field in crystals. It was then called the Inverse Faraday Effect (IFE). This magnetization follows from the laws of Biot-Savart and Ampere in electrodynamics and is determined with the help of the analytical formula for magnetic moment which gives the expression for the nonlinearly excited second order zero harmonic field in a wave affected plasmas. Since the loops of electric current for generation of this field need not necessarily be circular, the name IFE for the magnetic moment field has been avoided by us. The zero harmonic (DC) field of IFE, was first experimentally determined in presence of strong circularly polarized microwave radiation in some crystals [5–7]. In plasmas, the study of IFE by a circularly polarized wave was initiated by Pomeau and Quemada [8]. The influence of ion motion was considered by Chian [9]. Deschanps *et al* [10] experimentally first observed this effect in plasmas. Using nonfamiliar techniques of solution of the field equations the IFE was studied by Talin *et al* [11]. Considering the relativistic variation of electron mass, the nonlinearly induced magnetic moment density for a circularly polarized wave in an electron plasma was determined by Steiger and Woods [12]. Subsequently, Chakraborty and his coworkers [13–18] have developed the theoretical infrastructure for calculating the magnetic moment field in plasma under different physical situations particulary in laser induced plasma. In a review paper, Stamper [19] discussed some aspects of the generation of magnetic field in laser produced plasma.

It is felt that the theory of magnetic moment field may be further developed to understand the basic physics of the subject. So in this article, qualitative features and mathematical physics of basics of magnetic moment have been discussed keeping in mind that it would be applicable both in laser induced and astrophysical plasma. This work does provide the scope of study of new types of evolution of magnetic field which is not possible in the existing set up.

2. Basic relations

For a particle of mass m and charge q, the angular momentum $\boldsymbol{L}$ and the magnetic moment μ, in the CGS Gaussian units, are

$$\boldsymbol{L} = m(\boldsymbol{r} \times \dot{\boldsymbol{r}}),\ \mu = \frac{q}{2c}(\boldsymbol{r} \times \dot{\boldsymbol{r}}), \tag{1}$$

where $\boldsymbol{r}(t)$ is the position vector of the particle at time t and $\dot{\boldsymbol{r}}(t)$ is its velocity. In a material continuum, the dielectric polarization vector density $\boldsymbol{P}$ from transitional displacement and the magnetic moment density $\boldsymbol{M}$ from rotational displacement are given by

$$\boldsymbol{P} = N_0 q\xi,\ \boldsymbol{M} = \frac{1}{2c}(\xi \times \boldsymbol{j}), \tag{2}$$

where ξ is the field induced displacement, $\boldsymbol{j}$ is the induced electric current and N_0 is the number density of particles of charge q. The features of the excited moment field depend on the parameters of the applied wave field, and the thermal state existing and causing local thermal modes of energy transfer. For wave fields in plasma, this formula gives a DC (zero harmonic) moment field and a double harmonic moment field, both of second order of small quantities. This formula is suitable for finding zero harmonic moment $\boldsymbol{M}_0$ in problems of wave-plasma interactions and wave-wave interactions in plasma.

The integral (domain) formula for magnetic moment density in the neighbourhood of a point P for a charge q is

$$\mu = \frac{qA}{cT}\hat{n} \tag{3}$$

where A is the area of the loop described by q and T is the time of description of a complete loop (or the time period), and $\hat{n}$ is the unit vector normal to the plane of area A. In material media the corresponding density formula is

$$\boldsymbol{M} = \frac{1}{c}N_0 IA\hat{n}. \tag{4}$$

where I is the average volume current per loop and N_0 is the number density of the loops.

We now write

$$\boldsymbol{r} = \boldsymbol{r}_0 + \xi(\boldsymbol{r}_0, t), \tag{5}$$

where $\boldsymbol{r}_0$ is the position vectors of the point P and $\xi(\boldsymbol{r}_0, t)$ is the displacement induced by the applied wave field. Using (5) in (2) we obtain

$$\boldsymbol{M} = \frac{1}{2c}(\boldsymbol{r} \times \boldsymbol{j}) + \frac{1}{2c}(\xi \times \boldsymbol{j}), \tag{6}$$

where $\boldsymbol{r}_0$ is the pre-field position vector of the point P and $\xi(\boldsymbol{r}_0, t)$ is the displacement induced by the applied wave field. The components of $\boldsymbol{r}$ are independent variables. In the similar formula (2), in place of position vector $\boldsymbol{r}$, the field induced displacement vector ξ which is a function of the pre-field position vector $\boldsymbol{r}_0$ and time t, has been used. The time derivative of ξ is the velocity perturbation $\boldsymbol{u}$ where the time derivative of $\boldsymbol{u}$ is the acceleration in the equation of motion for a plasma continuum.

For several species of particles, we write

$$\boldsymbol{L}_0 = \sum_\alpha m_\alpha(\boldsymbol{r}_\alpha \times \dot{\boldsymbol{r}}_\alpha) = \sum_\alpha\left(\frac{2c}{q}m_\alpha\mu_\alpha\right) \tag{7}$$

for the angular momentum L_0

The magnetic moment M_0 is given by a

$$\sum \mu_\alpha = \boldsymbol{M}_0 = \sum_\alpha[q_\alpha(\boldsymbol{r}_\alpha \times \dot{\boldsymbol{r}}_\alpha)] \tag{8}$$

Hence, in a plasma continuum, consisting of several species of plasma :

$$\boldsymbol{M} = \frac{1}{2c}\sum_\alpha(\xi_\alpha \times \boldsymbol{j}_\alpha), \tag{9}$$

$$\boldsymbol{P} = \sum_\alpha(N_\alpha q_\alpha \xi_\alpha), \tag{10}$$

$$\boldsymbol{L} = \sum_\alpha N_\alpha q_\alpha(\xi_\alpha \times \dot{\xi}_\alpha), \tag{11}$$

$$\boldsymbol{j} = \sum(N_\alpha q_\alpha \boldsymbol{u}_\alpha), \tag{12}$$

where $\boldsymbol{u}_\alpha = \dot{\xi}_\alpha$, dot denotes time derivatives, N_α is the number density of α-th species of charged particles, *etc.* ξ_α is determined from the equation of motion for this species. These equations are valid for several species of charged fluids in presence of a single force field. However, in presence of several force fields, applied simultaneously in a multi species plasma, the nonrelativistic magnetic moment density and the polarization density are given by

$$\boldsymbol{P} = \sum_\alpha \boldsymbol{P}_\alpha = \sum_\alpha\left[N_\alpha q_\alpha\left(\sum_\beta \xi_\alpha^\beta\right)\right], \tag{13}$$

$$\boldsymbol{M} = \sum_\alpha \boldsymbol{M}_\alpha = \sum_\alpha\left[\frac{N_\alpha q_\alpha}{2c}\left(\sum_\beta \xi_\alpha^\beta \times \dot{\xi}_\alpha^\beta\right)\right], \tag{14}$$

$$\boldsymbol{j} = \sum_\alpha \boldsymbol{j}_\alpha = \sum_\alpha\left[N_\alpha q_\alpha\left(\sum_\beta \boldsymbol{u}_\alpha^\beta\right)\right]. \tag{15}$$

Here, the subscript β denotes the β-th force field, and the subscript α stands for the α-th species of particles. Hence, ξ_α^β is the displacement and $\boldsymbol{j}_\alpha^\beta$ is the electric current induced by the β-th force field on the particles of the α-th species of plasma.

Similarly, the expression for the volume current I and domain formula for M are

$$I = \sum_{\alpha}\sum_{\beta}(N_{\alpha}q_{\alpha}f^{\beta}), \tag{16}$$

$$M = \frac{1}{c}\sum_{\alpha}\sum_{\beta}\left(N_{\alpha}q_{\alpha}f^{\beta}A_{\alpha}^{\beta}\hat{n}_{\alpha}^{\beta}\right), \tag{17}$$

where $f^{\beta}(= \omega^{\beta}/2\pi$, the frequency of the β-th wave field in radians per second), is the oscillation number per second.

These general formulas, written intuitively, reduce to the standard forms for a single force and a single fluid species. Their derivation on a more firm physical foundation is awaited

3. Magnetic moment density in the Lorentz force for convection current

We find here the involvement of magnetic moment density M in the Lorentz force density from the convection current j_c using a formula linking the volume current I and the surface current j, considering the approximation that the areas of loops of electric charges and the field point P are so small that M and I can be regarded constants in the integration over these regions.

The Lorentz force density $(j_c \cdot B)/c$ occurs in the equations generating magnetic field in different cases of major classical studies, which are for the thermal gradient source, the dynamo source, the Hall current source and the diffusive field source with the help of generalized Ohm's law. Also, the equation of torque of all the terms of Ohm's contains a term of M The magnetic moment equation from the torque law, also contains, a term of the curl of the polarization density P which is also a known source of M. For some other sources (the ponderomotive force), the M-dependent term follows from an equation of an electron plasma.

From the definition of volume current I and surface current j of electrodynamics [20,21], we write

$$j_i\,dV = \sum^{N_0}(I_i dr_i) - I dr, \quad I = I_0 N_0, \tag{18}$$

where dV is the three-dimensional volume element, dr_i is the line element of the i-th loop carrying the current I_i, I is the average volume current density at the point $P(r)$, dr is the average of the line elements and N_0 is the number of current loops per unit volume. We also assume the small loop approximation; so that I and B remain constants in the areas of these loops. Hence, I and B are taken out of the integration used in the theoretical investigations. The convection current j_c is used in the Lorentz force density to find the M-dependent term with the help of (18). The Lorentz force term containing the conduction current j_c finds the Hall current.

With the help of eq. (18), we integrate the Lorentz force density F_p and obtain

$$\iiint_V F_p dV = \frac{1}{c}\iiint_V (j_c \times B)dV = \sum^{N_0}\oint_P \frac{1}{c}I_i^0(dr_i \times B)$$

$$= \sum^{N_0}\iint_{\Sigma_0}\frac{1}{c}I_0\left[\left(d\Sigma^0 \times \nabla\right)\times B\right], \tag{19}$$

where Stokes' theorem in the form

$$\oint_P dr \times B = \iint_{\Sigma_0}\left(d\Sigma^0 \times \nabla\right)\times B \tag{20}$$

for integrals, have been used; Σ_i^0 is the surface enclosing the i-th loop and $d\Sigma_i^0$ is a vector surface element at a point on I_i. Thus, we obtain the expression for the Lorentz force in terms of M :

$$F_p = (M \times \nabla)\times B\,. \tag{21}$$

Consider now the expression for the torque N of the Lorentz force

$$NdV = \frac{1}{c}(r \times (j \times B))dV = \frac{1}{c}(r \times (dr \times B))$$

$$= \frac{1}{c}\left[(r \cdot B)dr - (r \cdot dr)B\right]. \tag{22}$$

We assume that the variation B and I is negligible over the volume of a current loop Γ. Then the second term of the right side of (22) is an exact differential, integration of which over Γ vanishes. Then integrating (22), we obtain

$$\iiint_V NdV = \frac{1}{c}\oint_\Gamma (r \cdot B)dr. \tag{23}$$

Again, since $d((r \cdot B)r) = (r \cdot B)dr + (dr \cdot B)r$, the left side being an exact diferential, eq. (23) gives

$$\iiint_V NdV = -\frac{1}{c}\oint_\Gamma r(dr \cdot B). \tag{24}$$

Adding (23) and (24), we get

$$\iiint_V NdV = \frac{1}{2c}I\int_\Gamma[(r \cdot B)dr - (dr \cdot B)r]$$

$$= \frac{1}{2c}I\oint_\Gamma B \times (dr \times r) = \frac{1}{2c}\iiint_V B \times (j \times r)dV$$

$$= -\iiint_V (B \times M)dV. \tag{25}$$

Hence,

$$N = M \times B. \tag{26}$$

The results of (21) and (26) are known in physics, and the specific treatments of convection current in the Lorentz force term to obtain (21) and (26) are available in text book literature [20]. But the other steps to be adopted in what follows, are new and have not been considered earlier.

4. Magnetic moment in the ponderomotive force generating equation

Pondermotive force is the force of the gradient of the electric field amplitude which weakly varies along the direction of the wave path. This weak variation result is obtained after averaging over one time period $\frac{2\pi}{\omega}$ of that wave field. A strong laser force is responsible for the variation of the amplitude along the direction of the wave path. To find this force, we assume the electric field $E(\boldsymbol{r},t) = \boldsymbol{E}_0(\boldsymbol{r}_0 + \xi(\boldsymbol{r}_0,t)) \cdot \cos\theta$ for $\theta = \omega t$, and expand it in positive integral powers of the wave field induced displacement ξ, the partial time derivative of which is the velocity perturbation.

The relevant equations of motion of the electron plasma of the first and second order are

$$\dot{\boldsymbol{u}}_1 = -\frac{e}{m}E_1(\boldsymbol{r},t) = -\frac{e}{m}E_0(\boldsymbol{r}_0)\cos\theta, \tag{27}$$

$$\boldsymbol{u}_2 = -\frac{e}{m}E_2 + \frac{1}{mN_0}\left(j_1^0 \times \boldsymbol{B}_1\right). \tag{28}$$

In (28), we replace the first order current j_1^0 by the sum $j_c + j^p$ where $\boldsymbol{j}_c$ is the convection current of the Lorentz force and j^p is the ponderomotive force related current. Since $u_1 \quad \xi(\boldsymbol{r}_0, t)$, integration of (27) gives

$$\boldsymbol{u}_1 \doteq \xi = -\frac{e}{m\omega}\boldsymbol{E}(\boldsymbol{r}_0)\sin\theta,$$

$$-\frac{\quad}{m\omega^2}\boldsymbol{E}_0(\boldsymbol{r}_0)\cos\theta. \tag{29}$$

Then (28) can be written as

$$\boldsymbol{u}_2 = -\frac{e}{m}\cos\theta\left[\boldsymbol{E}_0(\boldsymbol{r}_0) + \frac{e}{m\omega^2}(\boldsymbol{E}_0\cdot\nabla_r)(\boldsymbol{E}_0\cos\theta)\right]$$

$$+\frac{1}{mNc}\left(j_c \times \boldsymbol{B}_1\right). \tag{30}$$

Faraday's law of EM induction gives the magnetic induction vector $\boldsymbol{B}_1$ in terms of $\boldsymbol{E}_1$. Thus,

$$\boldsymbol{B}_1 = -\frac{c}{\omega}\nabla_r \times \boldsymbol{E}_0(\boldsymbol{r}_0)\sin\theta. \tag{31}$$

Then the Lorentz force averaged over the wave time period $\frac{2\pi}{w}$ reads

$$\frac{1}{c}\left\langle(j_c \times \boldsymbol{B})\right\rangle = -\frac{Nc}{c}\left\langle\frac{c}{m\omega}\boldsymbol{E}_0(\boldsymbol{r}_0)\sin\theta\right.$$

$$\left.\times\left(-\frac{c}{\omega}\nabla_{r0}\times\boldsymbol{E}_0(\boldsymbol{r}_0)\sin\theta\right)\right\rangle$$

$$-\frac{Nc^2}{2m\omega^2}\left[\boldsymbol{E}_0(\boldsymbol{r}_0)\times(\nabla\times\boldsymbol{E}_0(\boldsymbol{r}_0))\right], \tag{32}$$

where $\langle x\rangle = \frac{1}{T}\int_0^T x\,dt$ for $T = \frac{2\pi}{\omega}$.

Hence using the vector formula $\nabla\left(\frac{A^2}{2}\right) = (\boldsymbol{A}\cdot\nabla)\boldsymbol{A} + \boldsymbol{A} \times (\nabla\times\boldsymbol{A})$, we find that

$$\langle\boldsymbol{u}_2\rangle = \frac{e}{2m^2\omega^2}(\boldsymbol{E}_0\cdot\nabla)\boldsymbol{E}_0$$

$$+\frac{1}{mcN_0}\left\langle j_c \times \boldsymbol{B}_1\right\rangle = \boldsymbol{F}_p + \boldsymbol{F}_M, \tag{33}$$

where $$\boldsymbol{F}_M = \frac{1}{mN_0}(\boldsymbol{M}\times\nabla)\times\boldsymbol{B}, \tag{34}$$

$$\boldsymbol{F}_p = -\nabla\frac{\quad}{4m^2\omega^2} \tag{35}$$

Evidently, $\boldsymbol{F}_p$ is the ponderomotive force per particle, and $\boldsymbol{F}_M$ is the magnetic moment-dependent force per particle. Comparing these two forces, we find that

$$\frac{|F_M}{|F_p} \quad \frac{\frac{1}{mN_0}(\boldsymbol{M}\times\nabla)\times\boldsymbol{B}}{\nabla|\frac{e^2E_0^2}{4m^2\omega^2}} = 16\pi\frac{l_p}{l_M}\frac{MB}{E_0^2}\frac{\omega^2}{\omega_{pe}^2} \tag{36}$$

where $\omega_{pe}^2 = \frac{4\pi N_0 e^2}{m}$; $|\nabla|$ of F_M is replaced by $\frac{1}{l_M}$ and $|\nabla|$ of F_p by $\frac{1}{l_p}$, so that l_M is the characteristic length of variation of F_M and l_p is the same of F_p.

In the presence of a circularly polarized wave of amplitude a, frequency ω and wave number k, in the linearized approximation of the field equation, the DC magnetic moment density is given by

$$M_z = -\frac{a^2c^3N}{\quad} \tag{37}$$

Hence, the relation (36) reads

$$\frac{|F_M|}{|F_p} - \frac{2ea}{mc\omega}. \tag{38}$$

This relation shows that as the field intensity increases, $|F_M|$ increases and $|F_p|$ decreases with increase of the applied wave frequency.

5. Magnetic moment in the generalized Ohm's law for fources of magnetization

For study of evolution of magnetization, we now consider the generalized Ohm's law of the form [23],

$$0 = -\frac{1}{N_0}(\nabla p_e) - e\boldsymbol{E} - \frac{e}{c}(\boldsymbol{u}\times\boldsymbol{B})$$

$$+\frac{1}{cN_0}(\boldsymbol{J}\times\boldsymbol{B}) + \frac{\nu_e\boldsymbol{J}}{eN_0} + \Pi \tag{39}$$

where $\boldsymbol{u}$ is the plasma stream velocity, $p_e = K_BN_eT_e$, K_B is the Boltzman constant, T_e is the electron kinetic temperature,

ν_e is the electron collision frequency, and Π represents the sum of the remaining terms, including the nonlinear terms of higher order. In the fourth term in the right hand side, we replace $\boldsymbol{j}$ by the sum $\boldsymbol{j}^c + \boldsymbol{j}_c$, where $\boldsymbol{j}_c = Nq\boldsymbol{u} = \sum_i (N_i q_i \boldsymbol{u}_i)$, $\boldsymbol{j}^c = \sigma \boldsymbol{E} + \frac{i\sigma}{c}(\boldsymbol{u} \times \boldsymbol{B})$. Then using the relation (21) for the Lorentz force density, we get

$$\boldsymbol{E} = \frac{1}{eN_e}\nabla P_e - \frac{1}{c}(\boldsymbol{u} \times \boldsymbol{B}) + \frac{1}{ecN_e}(\boldsymbol{j}^c \times \boldsymbol{B})$$

$$\frac{}{eN_0}(\boldsymbol{M} \times \nabla) \times \boldsymbol{B}. \tag{40}$$

Now we take curl of all the terms of this equation and applying Faraday's law of EM induction, obtain

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$= \frac{K_B}{cN_e}(\nabla T_e \times \nabla N_e) - \frac{1}{c}\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \frac{1}{cc}\nabla$$

$$\times \left[\frac{1}{N_e}(\boldsymbol{j}^c \times \boldsymbol{B})\right] + \frac{m\nu_e}{c^2}\nabla \times \left(\frac{1}{N_e}\boldsymbol{j}\right) + \frac{1}{c}\nabla$$

$$\frac{}{N_e}(\boldsymbol{M} \times \nabla) \times \boldsymbol{B}\Big]. \tag{41}$$

Hence, the magnetization evolution equation reads

$$\frac{\partial \boldsymbol{B}}{\partial t} + \frac{c}{e}\nabla \times \left[\frac{(\boldsymbol{M} \times \nabla) \times \boldsymbol{B}}{N_e}\right.$$

$$\frac{cK_B}{eN_e}(\nabla T_e \times \nabla N_e) + \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \frac{1}{c}\nabla$$

$$\frac{(\boldsymbol{j}_c \times \boldsymbol{B})}{N_e} \quad \frac{mc\nu_e}{e^2}\left[\nabla \times \left(\frac{1}{N_e}\boldsymbol{j}\right.\right. \tag{42}$$

The first term in the left side is the familiar time derivative of $\boldsymbol{B}$. The second term in the left-hand side gives the $\boldsymbol{M}$-dependent force per unit volume

$$\boldsymbol{F}_1 = \frac{e}{c}\nabla \times \left[\frac{1}{N_e}(\boldsymbol{M} \times \nabla) \times \boldsymbol{B}\right]. \tag{43}$$

It depends on space derivatives of $\boldsymbol{M}$. Evolution of the magnetostatic field is obtained by ignoring $\frac{\partial \boldsymbol{B}}{\partial t}$ in (42) and investigating the resulting equation which is independent of time :

$$\nabla \times \left[\frac{1}{N_e}(\boldsymbol{M} \times \nabla) \times \boldsymbol{B}\right]$$

$$= \frac{K_B}{N_e}(\nabla T_e \times \nabla N_e) + \frac{e}{c}\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \frac{1}{c}\nabla$$

$$\times \left[\frac{1}{N_e}(\boldsymbol{j}^c \times \boldsymbol{B})\right] - \frac{m\nu_e}{e}\nabla \times \left(\frac{\boldsymbol{j}}{N_e}\right). \tag{44}$$

It is to be mentioned that the first term on the right hand side of (2) represents the thermal gradient source, the second term represents the dynamo field, the third term gives the Hall current and the fourth term stands for the diffusive field.

6. Magnetic moment in the equation of the torque of all the terms of the generalized Ohm's law

The equation for the torque of all the terms of the generalized Ohm's law, which is in essence an equation of motion, is

$$(\boldsymbol{r} \times \boldsymbol{E}) = -\frac{1}{c}\left[\boldsymbol{r} \times \left(\frac{1}{N_e}\nabla P_e\right)\right] - \frac{1}{c}[\boldsymbol{r} \times (\boldsymbol{u} \times \boldsymbol{B})]$$

$$+ \frac{1}{cc}\boldsymbol{r} \times \left[\frac{1}{N_e}(\boldsymbol{j}^c \times \boldsymbol{B})\right] + \frac{1}{N_e}(\boldsymbol{M} \times \boldsymbol{B})$$

$$+ \frac{m\nu_e}{e^2}\ \boldsymbol{r} \times \left[\frac{1}{N_e}\boldsymbol{j}\right. \tag{45}$$

Let us take curl of (45) and use the vector relation

$$\nabla \times (\boldsymbol{r} \times \boldsymbol{A}) = -2\boldsymbol{A} + \boldsymbol{r}(\nabla \cdot \boldsymbol{A}) - (\boldsymbol{r} \cdot \nabla)\boldsymbol{A}$$

for further simplification and obtain

$$\boldsymbol{E} = -\frac{1}{eN_e}\nabla P_e - \frac{1}{c}(\boldsymbol{u} \times \boldsymbol{B}) + \frac{1}{ceN_e}(\boldsymbol{j}^c \times \boldsymbol{B})$$

$$\frac{m\nu_e}{e^2 N_e}\boldsymbol{j} - \frac{1}{2c}\nabla \times \left[\frac{1}{N_e}(\boldsymbol{M} \times \boldsymbol{B})\right] + 2\pi \boldsymbol{P}, \tag{46}$$

where $\boldsymbol{P}(= \boldsymbol{r}\rho^c$, ρ^c is the space charge separation per unit volume) is the polarization per unit volume. This equation contains the $\boldsymbol{M}$-dependent force per unit mass

$$\boldsymbol{F}_2 = \frac{e}{2c}\nabla \times \left[\frac{1}{N_e}(\boldsymbol{M} \times \boldsymbol{B})\right]. \tag{47}$$

Since it is the curl of a vector, it has space derivatives and it does not contain time derivatives. Also eq. (46) contains all the terms of eq. (39), and a polarization term which is new and some other terms of smaller orders of the type of those in the right hand side, which we have ignored. Taking curl of all the terms of equation (46), we obtain the magnetic field evolution equation

$$\frac{\partial \boldsymbol{B}}{\partial t} + \frac{c}{2e}\nabla \times \left[\nabla \times \frac{1}{N_e}(\boldsymbol{M} \times \boldsymbol{B})\right] + 2\pi c(\nabla \times \boldsymbol{P})$$

$$\frac{cK_B}{eN_e}(\nabla T_e \times \nabla N_e) + \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \frac{1}{c}\nabla$$

$$\times \left[\frac{1}{N_e}(\boldsymbol{j}^c \times \boldsymbol{B})\right] - \frac{mc\nu_e}{c^2}\nabla \times \left(\frac{1}{N_e}\boldsymbol{j}\right). \tag{48}$$

The left-hand side contains in addition to the usual time derivations of $\boldsymbol{B}$, a term depending on $\boldsymbol{M}$ and a new term

$2\pi c(\nabla \times \boldsymbol{P})$ which gives the magnetization from the curl of polarization.

Comparison of the order of magnitude of the dependent terms in the left side of (42) and (43) gives

$$\frac{|F_1|}{|F_2|} \quad \frac{\left|\frac{e}{c}\nabla\times\left[\frac{1}{N_e}(\boldsymbol{M}\times\nabla)\times\boldsymbol{B}\right]\right|}{\left|\frac{e}{2c}\nabla\times\left[\nabla\times\left(\frac{1}{N_e}(\boldsymbol{M}\times\boldsymbol{B})\right)\right]\right|} \tag{49}$$

where F_1 is given by (43) and F_2 is the second term in the left hand side of (48) given by (47).

7. Nonlinear Harmonic generation of magnetic moment

If L is the angular momentum density, then

$$L - N, \tag{50}$$

where N is the torque of the force density F. Since $\boldsymbol{M} = \Gamma\boldsymbol{L}$, where Γ is the gyromagnetic ratio tensor. For a single electron, $\Gamma = \frac{e}{2mc}$. Using (31) this relation gives

$$\boldsymbol{M} - \Gamma(\boldsymbol{M}\times\boldsymbol{B}) = -(\Omega^{\prime}_{M}\times\boldsymbol{B}), \tag{51}$$

where Ω^{e}_{M} is the electron magnetization gyrofrequency. More generally, including also ion dynamics, which is non-negligible for waves of low frequency (wave frequency less than the ion gyration frequency), we write

$$\boldsymbol{M}_e - -(\Omega^{\prime}_{M}\times\boldsymbol{H}), \boldsymbol{M}_i = -(\Omega^{\prime}_{M}\times\boldsymbol{H}), \tag{52}$$

where

$$\Omega^{e}_{M} - \frac{e}{mc}\left|\boldsymbol{M}_e + \frac{H_0}{4\pi}\right|, \Omega^{i}_{M} = -\frac{e}{mc}\left[\boldsymbol{M}_i + \frac{H_0}{4\pi}\right. \tag{53}$$

Solving (50) for a circularly polarized wave given by $\boldsymbol{E}$ - $a(\cos\theta, \sin\theta, 0)$, where θ - $kz - wt$, we obtain

$$\boldsymbol{M} - \boldsymbol{M}_e + \boldsymbol{M}_i = \frac{an}{w}(\Omega^{e}_{H} - \Omega^{i}_{H})(-\sin\theta, \cos\theta, 0) \tag{54}$$

for the first harmonic generation of the magnetic moment.

8. Four-vector formulation

The relativistically correct angular momentum of a system of N particles is

$$\boldsymbol{L} = \sum_i (m_i\gamma_i(\boldsymbol{r}_i \times \boldsymbol{v}_i)), \tag{55}$$

where m_i is the rest mass of the i-th particle, $\gamma_i (= (1-\beta_i^2)^{-\frac{1}{2}}$, $\beta_i = \left(\frac{\boldsymbol{u}_i}{c}\right)$ is the relativistic factor. The 4-vector representation of the angular momentum is

$$L^{\mu\nu} = \sum_i \left(x_i^{\mu} p_i^{\nu} - x_i^{\nu} p_i^{\mu}\right), \tag{56}$$

where

$$x_i^{\mu} = (\boldsymbol{r}_i, ct),\ p_i^{\mu} = \left(\boldsymbol{p}, \left(\frac{\xi_i}{c}\right)\right),\ \xi_i = \xi_i m_i c^2 = m_i c^2. \tag{57}$$

The 4-vector magnetic moment density $M^{\mu\nu}$ is similarly defined by

$$M^{\mu\nu} = \sum \frac{\left(x_i^{\mu} j_i^{\nu} - x_i^{\nu} j_i^{\mu}\right)}{2c} \tag{58}$$

where $j_i^{\mu} = (\boldsymbol{j}_i, c\rho^c)$, ρ^c is the charge density.

9. Some assessmental remarks

The magnetic moment physics will remain valid when the mean free path in the plasma is greater than the average loop diameter, because then collisions cannot effectively destroy the loops current for generation of the magnetic moment.

Classical theory of magnetic moment will fail and quantum mechanical theory will be necessary for X-rays and for magnetic fields in the range of tesla. This is because X-ray wave lengths are of the order of deBroglie wave lengths, and for tesla fields, the gyration radius of charges is of the order of atomic dimensions.

With the help of the electrodynamical relation $jdv = Idr$, and some vector relations, the Lorentz force density for the convection current of electricity is expressed in terms of $\boldsymbol{M}$ Also the torque of the same Lorentz force which appears in a magnetization evolution equation, is expressed in terms of $\boldsymbol{M}$. Moreover, the torque of the electrostatic force which also appears in this equation, generates a term of curl of the polarization density vector $\boldsymbol{P}$ as another source of magnetization.

Emphasis has been given both on the qualitative aspects and on the development of basic theory. In particular, the complementarity of magnetric moment and dielectric polarization has been discussed; the facts have been emphasized that magnetic moment has been shown to be a primary source of generation of magnetization; this field follows from the basic laws of electrodynamis; and consequenty from asymmetry of electric current and motion of charges. So, it is the basic process of generation of magnetostatic field. Hence, the existence of DC magnetic moment fields in wave affected, and unmagnetized plasma, challenges the astrophysical assumption of the primeval cosmic magnetic field for existence of present form of fields of space bodies, which are explained by the dynamo theories that require an *a priori* existence of a seed magnetic field.

Classical physics of magnetic moment has many interesting and useful aspects which remain undeveloped and unused. In this article, we cover nonrelativistically in an ordered manner, basic and preliminary aspects of this physics. The important formulas have been discussed and generalized intuitively for several species of charged fluids and several force fields acting simultaneously in a plasma.

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